UNIT:-I Chapter:-1

of proposition

Symbolic Logic and Algebra

Q1. Statement or proposition:-

=> An statement is a declarative sentence which is true or false, but not both; or in other words an statement is a declarative sentence which has a definite truth value

USE of Brackets:-

=> The use of bracket in statements (or proposition) is very important. The meaning (or explanation) of statements is included in brackets. for Example: The statements

$$(p \wedge q) \Rightarrow r \text{ and } p \wedge (q \Rightarrow r)$$

have completely different meaning. Therefore following rules are followed for brackets in statements.

connective word	non connective of word	Symbol
NOT	Negation	$\sim$
and	conjunction	$\wedge$
or	Disjunction	$\vee$
if	conditional	$\Rightarrow$
iff	Biconditional	$\Leftrightarrow$

1. He is a poor and laborious

$$\underset{p}{\downarrow} \qquad \underset{q}{\downarrow}$$

$\Rightarrow$ $(p \wedge q)$

2. He is a poor but is not laborious

$\Rightarrow$ $(p \wedge \sim q)$

3. It is not true that he is not poor or is not laborious

$\Rightarrow$ $\sim(\sim p \vee \sim q)$

4. Ram is rich = p q = Ram is happy

1. Ram is poor but happy

$\Rightarrow$ $\sim p \wedge q$

2. Ram is neither rich nor happy

$\Rightarrow$ $\sim p \sim \wedge q$

Truth Table :-

(A) Conjunction :- (∧)

(i) If p is true and q is true, then $p \wedge q$ is true
(ii) If p is true and q is false, then $p \wedge q$ is false
(iii) If p is false and q is true, then $p \wedge q$ is false
(iv) If p is false and q is false, then $p \wedge q$ is false

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

(B) Disjunction :- (∨)

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

CHAPTER ⇒ 1

SYMBOLIC LOGIC AND ALGEBRA OF PROPOSITION

* Kinds of sentences :-

i) Conjunctive Sentence $\wedge$

ii) Disjunctive Sentence $\vee$

iii) Conditional Sentence $\rightarrow$ or $\Rightarrow$

iv) Bi-conditional Sentence $\leftrightarrow$ or $\Leftrightarrow$

v) Negative Sentence $\sim$ or $\neg$

* Truth Table :-

1) Truth table for $p \wedge q$

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

2) Truth table for $p \vee q$

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

3) Truth table for $\sim p$

p	$\sim p$
T	F
F	T

4) Truth table for $p \Rightarrow q$

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

5) Truth table for $p \Leftrightarrow q$

p	q	$p \Leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

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* TAUTOLOGY :- A tautology is also called logically true

1) $(p \Leftrightarrow q) \wedge (q \Leftrightarrow r) \Rightarrow (p \Leftrightarrow r)$

p	q	r	$p \Leftrightarrow q$	$q \Leftrightarrow r$	$(p \Leftrightarrow q) \wedge (q \Leftrightarrow r) = A$	$p \Leftrightarrow r = B$	$A \Rightarrow B$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	F	F	T	T
T	F	F	F	T	F	F	T
F	T	T	F	T	F	F	T
F	T	F	T	F	F	T	T
F	F	T	T	F	F	F	T
F	F	F	T	T	T	T	T

2) $(\sim p) \wedge (p \Rightarrow q) \Rightarrow (\sim p)$

p	q	$\sim q$	$p \Rightarrow q$	$(\sim q) \wedge (p \Rightarrow q)$	$\sim p$	$(\sim q \wedge p \Rightarrow q) \Rightarrow (\sim p)$
T	T	F	T	F	F	T
T	F	T	F	F	F	T
F	T	F	T	F	T	T
F	F	T	T	T	T	T

Ex 1:- $p \Rightarrow p$ (if is a statement)

p	$p \Rightarrow p$
T	T
F	T

3) $(p \Rightarrow q \wedge r) \Rightarrow (\sim r \Rightarrow \sim p)$

p	q	r	$q \wedge r$	$p \Rightarrow (q \wedge r)$ $= A$	$\sim r$	$\sim p$	$\sim r \Rightarrow \sim p$ $= B$	$A \Rightarrow B$
T	T	T	T	T	F	F	T	T
T	T	F	F	F	T	F	F	T
T	F	T	F	F	F	F	T	T
T	F	F	F	F	T	F	F	T
F	T	T	T	T	F	T	T	T
F	T	F	F	T	T	T	T	T
F	F	T	F	T	F	T	T	T
F	F	F	F	T	T	T	T	T

Ex. 2) a) $p \wedge q \Rightarrow p$

p	q	$p \wedge q$	$p \wedge q \Rightarrow p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

b) $p \wedge q \Rightarrow q$

p	q	$p \wedge q$	$p \wedge q \Rightarrow q$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

Ex 3:) $p \vee (\sim p)$

p	$\sim p$	$p \vee \sim p$
T	F	T
F	T	T

Ex 4) i) $p \Rightarrow p \vee q$

ii) $q \Rightarrow (p \vee q)$

p	q	$p \vee q$	$p \Rightarrow p \vee q$	$q \Rightarrow p \vee q$
T	T	T	T	T
T	F	T	T	T
F	T	T	T	T
F	F	F	T	T

Ex 5) $p \wedge q \Rightarrow p \vee q$:-

p	q	$p \wedge q$	$p \vee q$	$p \wedge q \Rightarrow p \vee q$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

Ex 6.) $(\sim p) \vee q$

p	$\sim p$	q	$\sim p \vee q$
T	F	T	T
T	F	F	F
F	T	T	T
F	T	F	T

$\Rightarrow (\sim p) \vee q$ is not tautology.

Ex 7) $\sim(p \wedge (\sim p))$

p	$\sim p$	$p \wedge (\sim p)$	$\sim(p \wedge (\sim p))$
T	F	F	T
F	T	F	T

Ex 8) $p \Rightarrow (\sim p) \Leftrightarrow \sim p$

p	$\sim p$	$p \Rightarrow \sim p$	$p \Rightarrow (\sim p) \Leftrightarrow (\sim p)$
T	F	F	T
F	T	T	T

Ex. 9) $(p \Rightarrow q) \Leftrightarrow (\sim p \vee q)$

p	q	$\sim p$	$p \Rightarrow q$	$\sim p \vee q$	$p \Rightarrow q \Leftrightarrow (\sim p \vee q)$
T	T	F	T	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

Ex 10) $(p \Leftrightarrow q) \Leftrightarrow (p \Rightarrow q) \wedge (q \Rightarrow p)$

p	q	$p \Leftrightarrow q$	$p \Rightarrow q$	$q \Rightarrow p$	$(p \Rightarrow q) \wedge (q \Rightarrow p)$	$(p \Leftrightarrow q) \Leftrightarrow (p \Rightarrow q) \wedge (q \Rightarrow p)$
T	T	T	T	T	T	T
T	F	F	F	T	F	T
F	T	F	T	F	F	T
F	F	T	T	T	T	T

Ex 11) $(p \Rightarrow q) \wedge (q \Rightarrow p) \Leftrightarrow (p \Leftrightarrow q)$

p	q	$p \Rightarrow q$	$q \Rightarrow p$	$(p \Rightarrow q) \wedge (q \Rightarrow p) = P$	$P \Leftrightarrow q$ = Q	$P \Leftrightarrow Q$
T	T	T	T	T	T	T
T	F	F	T	F	F	T
F	T	T	F	F	F	T
F	F	T	T	T	T	T

Ex. 12) $(p \Leftrightarrow q) \wedge (q \Leftrightarrow r) \Rightarrow (p \Leftrightarrow r)$

p	q	r	$p \Leftrightarrow q$	$q \Leftrightarrow r$	$p \Leftrightarrow r = P$	$(p \Leftrightarrow q) \wedge (q \Leftrightarrow r) = B$	$B \Rightarrow A$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	F	T	F	T
T	F	F	F	T	F	F	T
F	T	T	F	T	F	F	T
F	T	F	F	F	T	F	T
F	F	T	T	F	F	F	T
F	F	F	T	T	T	T	T

Ex 13) ii) $[(\sim q \Rightarrow \sim p) \wedge (q \Rightarrow p)] \Rightarrow (p \Leftrightarrow q)$. — P, Q, R

p	q	$\sim p$	$\sim q$	$\sim q \Rightarrow \sim p$	$q \Rightarrow p$	$(\sim q \Rightarrow \sim p) \wedge (q \Rightarrow p)$ P Q	$p \Leftrightarrow q$ R	$(P \wedge Q) \Rightarrow R$
T	T	F	F	T	T	T	T	T
T	F	F	T	F	T	F	F	T
F	T	T	F	T	F	F	F	T
F	F	T	T	T	T	T	T	T

i) $[(p \Rightarrow q) \wedge (q \Rightarrow r)] \Rightarrow (p \Rightarrow r)$

p	q	r	$p \Rightarrow q$ P	$q \Rightarrow r$ Q	$p \Rightarrow r$ R	$P \wedge Q$	$(P \wedge Q) \Rightarrow R$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	T	F	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

iii) $(p \Rightarrow q) \vee (r \Rightarrow p)$

p	q	r	$p \Rightarrow q$	$r \Rightarrow p$	$(p \Rightarrow q) \vee (r \Rightarrow p)$
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	F	T	T
T	F	F	F	T	T
F	T	T	T	F	T
F	T	F	T	T	T
F	F	T	T	F	T
F	F	F	T	T	T

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iv) $(p \Leftrightarrow q \wedge r) \Rightarrow (\sim r \Rightarrow \sim p)$

p	q	r	$q \wedge r$	$p \Leftrightarrow q \wedge r$ (A)	$\sim p$	$\sim r$	$\sim r \Rightarrow \sim p$ (B)	$A \Rightarrow B$
T	T	T	T	T	F	F	T	T
T	T	F	F	F	F	T	F	T
T	F	T	F	F	F	F	T	T
T	F	F	F	F	F	T	F	T
F	T	T	T	F	T	F	T	T
F	T	F	F	T	T	T	T	T
F	F	T	F	T	T	F	T	T
F	F	F	F	T	T	T	T	T

Ex 14)

i) $(p \Rightarrow q) \vee r \Leftrightarrow [(p \vee r) \Rightarrow (q \vee r)]$

p	q	r	$p \Rightarrow q$	$(p \Rightarrow q) \vee r$ (A)	$p \vee r$	$q \vee r$	$(p \vee r) \Rightarrow (q \vee r)$ (B)	$A \Leftrightarrow B$
T	T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T	T
T	F	T	F	T	T	T	T	T
T	F	F	F	F	T	F	F	F
F	T	T	T	T	T	T	T	T
F	T	F	T	T	F	T	T	T
F	F	T	T	T	T	T	T	T
F	F	F	T	F	F	F	T	T

ii) $(p \vee q \vee r) \Leftrightarrow [(((p \Rightarrow q) \Rightarrow q) \Rightarrow r) \Rightarrow r]$

p	q	r	$p \vee q \vee r$ $(=P)$	$p \Rightarrow q$	$p \vee q \Rightarrow q$	$((p \Rightarrow q) \Rightarrow q) \Rightarrow r$	$(((p \Rightarrow q) \Rightarrow q) \Rightarrow r) \Rightarrow r$ $= (Q)$	$P \Leftrightarrow Q$
T	T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T	T
T	F	T	T	F	T	T	T	T
T	F	F	T	F	T	F	T	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	T	F	T	T
F	F	T	T	T	F	T	T	T
F	F	F	F	T	F	T	F	T

iv) $(p \Rightarrow q) \Leftrightarrow (\sim p \vee q)$

p	q	$p \Rightarrow q$	$\sim p$	$\sim p \vee q$	$(p \Rightarrow q) \Leftrightarrow (\sim p \vee q)$
T	T	T	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

vi) $(p \wedge q \Rightarrow$

p	q	r	$p \wedge$
T	T	T	T
T	T	F	T
T	F	T	F
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	F

vii) $(p \Rightarrow q) \wedge$

p	q	r	p
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

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vi) $(p \wedge q \Rightarrow r) \Leftrightarrow (p \Rightarrow r) \vee (q \Rightarrow r)$

p	q	r	$p \wedge q$	$p \wedge q \Rightarrow r$ (P)	$p \Rightarrow r$	$q \Rightarrow r$	$(p \Rightarrow r) \vee (q \Rightarrow r)$ (Q)	$P \Leftrightarrow Q$
T	T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F	T
T	F	T	F	T	T	T	T	T
T	F	F	F	T	F	T	T	T
F	T	T	F	T	T	T	T	T
F	T	F	F	T	T	F	T	T
F	F	T	F	T	T	T	T	T
F	F	F	F	T	T	T	T	T

vii) $(p \Rightarrow q) \wedge (q \Rightarrow r) \Rightarrow (p \Rightarrow r)$

p	q	r	$p \Rightarrow q$	$q \Rightarrow r$	$(p \Rightarrow q) \wedge (q \Rightarrow r)$ (P)	$p \Rightarrow r$ (Q)	$P \Rightarrow Q$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

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* Contradiction:- A contradiction is also called logically false.

Ex 1) $p \wedge (\sim p)$

p	$\sim p$	$p \wedge (\sim p)$
T	F	F
F	T	F

Ex. 2) $p \Leftrightarrow (\sim p)$

p	$\sim p$	$p \Leftrightarrow (\sim p)$
T	F	F
F	T	F

Ex. 3) $(p \vee q) \wedge (\sim q) \wedge (\sim q)$

p	q	$p \vee q$	$\sim p$	$\sim q$	$(p \vee q) \wedge (\sim p)$ (A)	$A \wedge (\sim q)$
T	T	T	F	F	F	F
T	F	T	F	T	F	F
F	T	T	T	F	T	F
F	F	F	T	T	F	F

Ex 4) i) $P = [(p \vee q) \wedge (p \vee \sim q) \wedge (\sim p \vee q) \wedge (\sim p \vee \sim q)]$

p	q	$\sim p$	$\sim q$	$p \vee q$ R	$p \vee \sim q$ S	$\sim p \vee q$ U	$\sim p \vee \sim q$ V	$P = R \wedge S \wedge U \wedge V$
T	T	F	F	T	T	T	F	F
T	F	F	T	T	T	F	T	F
F	T	T	F	T	F	T	T	F
F	F	T	T	F	T	T	T	F

ii) $[(p \wedge r) \vee$ …

$(p$ …

p	q	r	$\sim$ …
T	T	T	F
T	T	F	F
T	F	T	F
T	F	F	F
F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	T

iii) $(p \vee q)$ …

p	q	$\sim p$
T	T	F
T	F	F
F	T	T
F	F	T

iv) $[(p \wedge q) \Rightarrow$ …

p q $\sim p$ $\sim$

p	q	$\sim q$
T	T	F
T	F	T
F	T	F
F	F	T

ii) $[(p \wedge r) \vee (q \wedge \sim r)] \Leftrightarrow [(\sim p \wedge r) \vee (\sim q \wedge \sim r)]$

(P) (Q)

p	q	r	~p	~q	~r	p∧r	q∧~r	P	~p∧r	~q∧~r	Q	P⟺Q
T	T	T	F	F	F	T	F	T	F	F	F	F
T	T	F	F	F	T	F	T	T	F	F	F	F
T	F	T	F	T	F	T	F	T	F	F	F	F
T	F	F	F	T	T	F	F	F	F	T	T	F
F	T	T	T	F	F	F	F	F	T	F	T	F
F	T	F	T	F	T	F	T	T	F	F	F	F
F	F	T	T	T	F	F	F	F	T	F	T	F
F	F	F	T	T	T	F	F	F	F	T	T	F

iii) $(p \vee q) \wedge (\sim p) \wedge (\sim q)$

p	q	~p	~q	p∨q	(p∨q)∧(~p)	(p∨q)∧(~p)∧(~q)
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	T	F
F	F	T	T	F	F	F

iv) $[(p \wedge q) \Rightarrow p] \Rightarrow [q \wedge \sim q]$

p q ~p ~q

p	q	~q	p∧q	(p∧q)⇒p	q∧~q	[(p∧q)⇒p]⇒(q∧~q)
T	T	F	T	T	F	F
T	F	T	F	T	F	F
F	T	F	F	T	F	F
F	F	T	F	T	F	F

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v) $(p \wedge q \Rightarrow q) \Rightarrow (q \wedge \sim q)$

p	q	$p \wedge q$	$(p \wedge q) \Rightarrow q$	$\sim q$	$q \wedge \sim q$	$[(p \wedge q) \Rightarrow q] \Rightarrow (q \wedge \sim q)$
T	T	T	T	F	F	F
T	F	F	T	T	F	F
F	T	F	T	F	F	F
F	F	F	T	T	F	F

** LOGICAL EQUIVALENCE :-

Two statements are called logically equivalent if the truth values of both the statements are always identical.

Ex :⇒ $x = 4$, $3x = 12$

$x = \frac{12}{3} = 4$

$x = 4$

Ex 1 ⇒ Show that implication $p \Rightarrow q$ & its contrapositive $(\sim q) \Rightarrow (\sim p)$ are logically equivalent.

p	q	$p \Rightarrow q$	$\sim p$	$\sim q$	$(\sim q) \Rightarrow (\sim p)$
T	T	T	F	F	T
T	F	F	F	T	F
F	T	T	T	F	T
F	F	T	T	T	T

The entries of 3^{rd} and 6^{th} columns are indentical. Hence $p \Rightarrow q$ and $(\sim q) \Rightarrow (\sim p)$ are logically equivalent.

Ex 2 ⇒ $q \Rightarrow p$
$(\sim p) \Rightarrow$

p
T
T
F
F

∴ (

Ex. 3 ⇒ $\sim (p$

p	q
T	T
T	F
F	T
F	F

∴ 5

Hence

Ex. 4 ⇒ $p \Rightarrow$

p	q	r
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

5th

∴ F

Ex 2 ⇒ $q \Rightarrow p$ converse of $p \Rightarrow q$ and its inverse $(\sim p) \Rightarrow (\sim q)$ are logically equivalent.

p	q	$q \Rightarrow p$	$\sim p$	$\sim q$	$(\sim p) \Rightarrow (\sim q)$
T	T	T	F	F	T
T	F	T	F	T	T
F	T	F	T	F	F
F	F	T	T	T	T

$\therefore (q \Rightarrow p) \equiv \{(\sim p) \Rightarrow (\sim q)\}$

Ex. 3 ⇒ $\sim (p \Rightarrow q) \equiv \{p \wedge (\sim q)\}$

p	q	$\sim q$	$p \Rightarrow q$	$\sim (p \Rightarrow q)$	$p \wedge (\sim q)$	$\sim(p \Rightarrow q) \Leftrightarrow (p \wedge (\sim q))$
T	T	F	T	F	F	T
T	F	T	F	T	T	T
F	T	F	T	F	F	T
F	F	T	T	F	F	T

$\therefore$ 5th and 6th collumns are indential.
Hence $\sim (p \Rightarrow q) \equiv \{p \wedge (\sim q)\}$

Ex. 4 ⇒ $p \Rightarrow (q \Rightarrow r) \equiv (p \wedge q) \Rightarrow r$

p	q	r	$q \Rightarrow r$	$p \Rightarrow (q \Rightarrow r)$	$p \wedge q$	$(p \wedge q) \Rightarrow r$	$p \Rightarrow (q \Rightarrow r) \Leftrightarrow (p \wedge q) \Rightarrow$
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	T
T	F	T	T	T	F	T	T
T	F	F	T	T	F	T	T
F	T	T	T	T	F	T	T
F	T	F	F	T	F	T	T
F	F	T	T	T	F	T	T
F	F	F	T	T	F	T	T

5th and ~~6th~~ 7th calums are indential.
$\therefore p \Rightarrow (q \Rightarrow r) \equiv (p \wedge q) \Rightarrow r$

Ex5)

i) $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

p	q	r	$p \wedge q$	$q \wedge r$	$(p \wedge q) \wedge r$ (P)	$p \wedge (q \wedge r)$ (Q)	$P \Leftrightarrow Q$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	F	F	F	T
T	F	F	F	F	F	F	T
F	T	T	F	T	F	F	T
F	T	F	F	F	F	F	T
F	F	T	F	F	F	F	T
F	F	F	F	F	F	F	T

6^{th} and 7^{th} columns are idential.

$\therefore (p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

ii) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

p	q	r	$p \wedge q$	$p \wedge r$	$q \vee r$	$p \wedge (q \vee r)$ (P)	$(p \wedge q) \vee (p \wedge r)$ (Q)	$P \Leftrightarrow Q$
T	T	T	T	T	T	T	T	T
T	T	F	T	F	T	T	T	T
T	F	T	F	T	T	T	T	T
T	F	F	F	F	F	F	F	T
F	T	T	F	F	T	F	F	T
F	T	F	F	F	T	F	F	T
F	F	T	F	F	T	F	F	T
F	F	F	F	F	F	F	F	T

Since $P \Leftrightarrow Q$ is tautology, we have

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

iii) $p \vee$

p	q
T	T
T	T
T	F
T	F
F	T
F	T
F	F
F	F

Since

iv) $p =$

p	q
T	T
T	T
T	F
T	F
F	T
F	T
F	F
F	F

iii) $p \vee (q \wedge r) = P$ and $(p \vee q) \wedge (p \vee r) = Q$

p	q	r	$p \vee q$	$p \vee r$	$q \wedge r$	P	Q	$P \Leftrightarrow Q$
T	T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T	T
T	F	T	T	T	F	T	T	T
T	F	F	T	T	F	T	T	T
F	T	T	T	T	T	T	T	T
F	T	F	T	F	F	F	F	T
F	F	T	F	T	F	F	F	T
F	F	F	F	F	F	F	F	T

Since $P \Leftrightarrow Q$ is tautology, we have
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

iv) $p \Rightarrow (q \wedge r) = P$ and $(p \Rightarrow q) \wedge (p \Rightarrow r) = Q$

p	q	r	$q \wedge r$	$p \Rightarrow q$	$p \Rightarrow r$	P	Q	$P \Leftrightarrow Q$
T	T	T	T	T	T	T	T	T
T	T	F	F	T	F	F	F	T
T	F	T	F	F	T	F	F	T
T	F	F	F	F	F	F	F	T
F	T	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T	T
F	F	T	F	T	T	T	T	T
F	F	F	F	T	T	T	T	T

Since $P \Leftrightarrow Q$ is tautology, we have

$p \Rightarrow (q \wedge r) \equiv (p \Rightarrow q) \wedge (p \Rightarrow r)$

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v) $p \Rightarrow (q \vee r) = P$ and $(p \Rightarrow q) \vee (p \Rightarrow r) = Q$

p	q	r	$q \vee r$	$p \Rightarrow q$	$p \Rightarrow r$	P	Q	$P \Leftrightarrow Q$
T	T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T	T
T	F	T	T	F	T	T	T	T
T	F	F	F	F	F	F	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T	T
F	F	T	T	T	T	T	T	T
F	F	F	F	T	T	T	T	T

Since $P \Leftrightarrow Q$ is tautology, we have
$p \Rightarrow (q \vee r) \equiv (p \Rightarrow q) \vee (p \Rightarrow r)$

vi) $(p \Rightarrow q) \vee r = P$ and $(p \vee r) \Rightarrow (q \vee r) = Q$

p	q	r	$p \vee r$	$q \vee r$	$p \Rightarrow q$	P	Q	$P \Leftrightarrow Q$
T	T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T	T
T	F	T	T	T	F	T	T	T
T	F	F	T	F	F	F	F	T
F	T	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T	T
F	F	T	T	T	T	T	T	T
F	F	F	F	F	T	T	T	T

$P \Leftrightarrow Q$ is tautology $\therefore (p \Rightarrow q) \vee r \equiv (p \vee r) \Rightarrow (q \vee r)$

vii) $\sim(p \wedge q)$ and $(\sim p) \wedge (\sim q)$

p	q	$p \vee q$	$\sim(p \vee q)$	$\sim p$	$\sim q$	$\sim p \wedge \sim q$	$\sim(p \vee q) \Leftrightarrow (\sim p) \wedge (\sim q)$
T	T	T	F	F	F	F	T
T	F	T	F	F	T	F	T
F	T	T	F	T	F	F	T
F	F	F	T	T	T	T	T

Since $\sim(p \vee q) \Leftrightarrow \sim p \wedge \sim q$ is a tautology $\therefore \sim(p \vee q) \equiv \sim p \wedge \sim q$

viii)

p
T
T
F
F

* * *

1)

i)

ii)

i)

ii)

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viii) $\sim(p \wedge q) \equiv \sim p \vee \sim q$

p	q	$p \wedge q$	$P = \sim(p \wedge q)$	$\sim p$	$\sim q$	$Q = \sim p \vee \sim q$	$P \Leftrightarrow Q$
T	T	T	F	F	F	F	T
T	F	F	T	F	T	T	T
F	T	F	T	T	F	T	T
F	F	F	T	T	T	T	T

Since $P \Leftrightarrow Q$ is tautology.
$\therefore \sim(p \wedge q) \equiv \sim p \vee \sim q$

* * * Algebra of proposition :-

1) Idemptotent laws :-

i) $P \vee P \Leftrightarrow P$
ii) $P \wedge P \Leftrightarrow P$

i) Truth table for $P \vee P \Leftrightarrow P$

P	P	$P \vee P$	$P \vee P \Leftrightarrow P$
T	T	T	T
F	F	F	T

$\therefore$ it is tautology.

ii) Truth table for $P \wedge P \Leftrightarrow P$

P	P	$P \wedge P$	$P \wedge P \Leftrightarrow P$
T	T	T	T
F	F	F	T

$\therefore$ it is tautology.

$A + B = B + A$
$A \cdot B = B \cdot A$

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2) Commutative laws :-

i) $p \vee q = q \vee p$
ii) $p \wedge q = q \wedge p$

i) Truth table of $p \vee q = q \vee p$

p	q	$p \vee q$	$q \vee p$	$p \vee q = q \vee p$
T	T	T	T	T
F	F	T	T	T
F	T	T	T	T
F	F	F	F	T

$\therefore$ it is tautology.

ii) Truth table of $p \wedge q = q \wedge p$

p	q	$p \wedge q$	$q \wedge p$	$p \wedge q = q \wedge p$
T	T	T	T	T
T	F	F	F	T
F	T	F	F	T
F	F	F	F	T

$\therefore$ it is tautology.

3) Associative laws :- $(a+b)+c = a+(b+c)$

i) $(p \vee q) \vee r \Leftrightarrow p \vee (q \vee r)$

ii) $(p \wedge q) \wedge r \Leftrightarrow p \wedge (q \wedge r)$

Proof. i)

p	q	r	$p \vee q$	(A) $(p \vee q) \vee r$	$q \vee r$	(B) $p \vee (q \vee r)$	$A \Leftrightarrow B$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	T	T	T	T	T
T	F	F	T	T	F	T	T
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	F	T	F	T	T	T	T
F	F	F	F	F	F	F	T

ii)

p	q	r	$p \wedge q$	(A) $(p \wedge q) \wedge r$	$q \wedge r$	(B) $p \wedge (q \wedge r)$	$A \Leftrightarrow B$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	F	F	F	T
T	F	F	F	F	F	F	T
F	T	T	F	F	F	F	T
F	T	F	F	F	F	F	T
F	F	T	F	F	F	F	T
F	F	F	F	F	F	F	T

4) Distributive laws :- $(a.(b+c) = (a.b + a.c))$

i) $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$

ii) $p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$

i)

p	q	r	$q\wedge r$	$p\vee q$	$p\vee r$	A: $p\vee(q\wedge r)$	B: $(p\vee q)\wedge(p\vee r)$	$A\Leftrightarrow B$
T	T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T	T
T	F	T	F	T	T	T	T	T
T	F	F	F	T	T	T	T	T
F	T	T	T	T	T	T	T	T
F	T	F	F	T	F	F	F	T
F	F	T	F	F	T	T	T	T
F	F	F	F	F	F	F	F	T

$\therefore$ it is tautology.

ii)

p	q	r	$q\vee r$	$p\wedge q$	$p\wedge r$	$p\wedge(q\vee r)$ (A)	$(p\wedge q)\vee(p\wedge r)$ (B)	$A\Leftrightarrow B$
T	T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T	T
T	F	T	T	F	T	T	T	T
T	F	F	F	F	F	F	F	T
F	T	T	T	F	F	F	F	T
F	T	F	T	F	F	F	F	T
F	F	T	T	F	F	F	F	T
F	F	F	F	F	F	F	F	T

Since the columns under $p\wedge(q\vee r)$ and $(p\wedge q)\vee(p\wedge r)$ are identical. Hence this is a tautology.